International Journal of Mathematics and
Computer Applications Research (IJMCAR)
ISSN (P): 2249-6955; ISSN (E): 2249-8060
Vol. 8, Issue 1, Feb 2018, 11-24

TRANS STELLAR •Journal Publications •Research Consultancy

SOME NEW EXACT TRAVELING WAVE SOLUTIONS OF THE mKdV EQUATION BYKHATER METHOD

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ABSTRACT

In this paper, Khater method is used to construct new analytical solutions of mKdV equation. As a result, some new types of exact traveling wave solutions are obtained using trigonometric, hyperbolic, exponential functions and rational forms. The related results are extended and the obtained results clearly indicate the reliability and efficiency of the Khater method.

KEYWORDS: Traveling Wave Solutions, Khater Method, & mKdV Equation

Received: Jan 04, 2018; **Accepted:** Jan 25, 2018; **Published:** Feb 14, 2018; **Paper Id.:** IJMCARFEB20182

Original Article

1. INTRODUCTION

To understand the nonlinear physical phenomena, arises in nature and in the field of applied sciences such as fluid dynamics, plasma physics, solid state physics, optical fibers, acoustics, mechanics, biology and mathematical finance, finding exact solution to nonlinear equation is important [1]. In nonlinear science, exact solutions to nonlinear partial differential equations play a great role, since they can provide much physical information and more insight of the physical aspects of the problem and thus lead to further applications. Wave phenomena in dispersion, dissipation, diffusion, reaction and convection are very much important. And many kinds of effective methods have been used to obtain explicit traveling and solitary wave solutions of nonlinear evolution equations have been established, such as, the ansatz method [2], the Adomian decomposition method [3], the Darboux transformation method [4], the Backlund transformation method [5], the inverse scattering transformation method [6], the Jacobi elliptic function method [7,8], the Exp-function method [9,10], the extended tanh method [11], the Cole-Hopf transformation [12], the (G'/G)-expansion method [13–17], and the modified simple equation method [18,19].

Very recent Khater et al [20] established a highly effective method called the Khater method to get exact and solitary wave solution of nonlinear partial differential equations (NPDEs.). This method is among one of those general methods that rely on auxiliary equation and in addition, comprise seven methods namely (G'/G)-expansion method[21], improved (G'/G)-expansion method [22], $\exp(-\phi(\xi))$ method [23], extended tanh-function method [24], kudryashov and modified kudryashov methods [25], improved $\tan\left(\varphi/2\right)$-expansion method [26], and novel (G'/G)-expansion method [27] subject to certain conditions which makes it an effective and powerful method. The aim of this work is to search for new study relating to the Khater method for solving the

mKdV equation to demonstrate the appropriateness and straightforwardness of the method.

The organization of the paper is as follows: in section 2, a brief description of Khater method is given. In section 3, we will study the mKdV equation by the new methods and in section 4 discussion on the result are provided. Finally conclusions are given in section 5followed by references in section ‘‘References”.

2. SUMMARY OF KHATER METHOD

In 2017, Khater method was first proposed by Khater et al [20] for solving nonlinear partial differential equations. Firstly, we consider a general nonlinear evaluation equation in two independent variables, say x and t,

$$\varphi(u, u_x, u_{t,} u_{xx,} u_{tt,} u_{xt}, \ldots\ldots\ldots) = 0, \tag{2.1}$$

Where $u = u(x,t)$ is an unknown function to be determined, φ is a polynomial of $u(x,t)$ and its partial derivatives wherein the highest order partial derivatives and the nonlinear terms are involved and the subscripts indicate the partial derivatives. The key steps of the method are as follows:

Step 1: We now introduce the traveling wave variable,

$$\xi = x - vt,\ u = u(x,t) = u(\xi), \tag{2.2}$$

where v is the speed of traveling wave and the wave variable (2.2) transforms Equation (2.1) into the following ordinary differential equation (ODE):

$$\phi(u, u', u'', u''', \ldots\ldots\ldots\ldots) = 0, \tag{2.3}$$

where ϕ is a polynomial of u and its derivatives and the super scripts stipulate the ordinary derivatives with respect to ξ.

Step 2: In many instances, Equation (2.3) can be integrated term by term one or more times, yielding constants of integration, which can be set equal to zero for straight forwardness.

Step 3: The Exact solution of Eq. (2.3) is supposed as

$$u(\xi) = \sum_{i=0}^{N} a_i a^{f(\xi)}, \tag{2.4}$$

where a_i, a, is constant to be determined, such that $a_N \neq 0$ and $f(\xi)$ satisfies the following second order linear ordinary differential equation (LODE):

$$f'(\xi) = \frac{1}{\ln(a)}(\alpha a^{-f(\xi)} + \beta + \sigma a^{f(\xi)}), \tag{2.5}$$

Step 4: The positive integer N can be determined by using the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in (2.3).

Step 5: Using Eq. (2.4) and Eq. (2.5) into Eq. (2.3), collecting all the terms having same powers of $a^{if(\xi)}$,

where $(i = 0, 1, 2, \ldots\ldots)$ and equating to zero, weform a system of algebraic equations. This system is further solved symbolically, to determine the values of a^i, α, β and σ.

The solutions of Eq. (2.5) subject to the coupled cases are given as follows

If $(\beta^2 - \alpha\sigma < 0 \,\&\, \sigma \neq 0)$.

$$a^{f(\xi)} = \left[-\frac{\beta}{\sigma} + \frac{\sqrt{-(\beta^2 - \alpha\sigma)}}{\sigma} \tan\left(\frac{\sqrt{-(\beta^2 - \alpha\sigma)}}{2} \xi \right) \right], \tag{2.6}$$

or

$$a^{f(\xi)} = \left[-\frac{\beta}{\sigma} + \frac{\sqrt{-(\beta^2 - \alpha\sigma)}}{\sigma} \cot\left(\frac{\sqrt{-(\beta^2 - \alpha\sigma)}}{2} \xi \right) \right]. \tag{2.7}$$

If $(\beta^2 - \alpha\sigma > 0 \,\&\, \sigma \neq 0)$.

$$a^{f(\xi)} = \left[-\frac{\beta}{\sigma} - \frac{\sqrt{(\beta^2 - \alpha\sigma)}}{\sigma} \tanh\left(\frac{\sqrt{-(\beta^2 - \alpha\sigma)}}{2} \xi \right) \right], \tag{2.8}$$

or

$$a^{f(\xi)} = \left[-\frac{\beta}{\sigma} - \frac{\sqrt{(\beta^2 - \alpha\sigma)}}{\sigma} \coth\left(\frac{\sqrt{-(\beta^2 - \alpha\sigma)}}{2} \xi \right) \right]. \tag{2.9}$$

If $(\beta^2 + \alpha^2 > 0 \,\&\, \sigma \neq 0 \,\&\, \sigma = -\alpha)$.

$$a^{f(\xi)} = \left[\frac{\beta}{\sigma} + \frac{\sqrt{(\beta^2 + \alpha^2)}}{\alpha} \tanh\left(\frac{\sqrt{(\beta^2 + \alpha^2)}}{2} \xi \right) \right], \tag{2.10}$$

or

$$a^{f(\xi)} = \left[\frac{\beta}{\sigma} + \frac{\sqrt{(\beta^2 + \alpha^2)}}{\alpha} \coth\left(\frac{\sqrt{(\beta^2 + \alpha^2)}}{2} \xi \right) \right]. \tag{2.11}$$

If $(\beta^2 + \alpha^2 < 0 \,\&\, \sigma \neq 0 \,\&\, \sigma = -\alpha)$.

$$a^{f(\xi)} = \left[\frac{\beta}{\sigma} + \frac{\sqrt{-(\beta^2 + \alpha^2)}}{\alpha} \tan\left(\frac{\sqrt{-(\beta^2 + \alpha^2)}}{2} \xi \right) \right], \tag{2.12}$$

or

$$ {}_{a}f(\xi) = \left[\frac{\beta}{\sigma} + \frac{\sqrt{-(\beta^2+\alpha^2)}}{\alpha} \cot\left(\frac{\sqrt{-(\beta^2+\alpha^2)}}{2} \xi \right) \right]. \tag{2.13} $$

If $(\beta^2 - \alpha^2 < 0 \,\&\, \sigma = \alpha)$.

$$ {}_{a}f(\xi) = \left[\frac{-\beta}{\sigma} + \frac{\sqrt{-(\beta^2-\alpha^2)}}{\alpha} \tan\left(\frac{\sqrt{-(\beta^2-\alpha^2)}}{2} \xi \right) \right], \tag{2.14} $$

or

$$ {}_{a}f(\xi) = \left[\frac{-\beta}{\sigma} + \frac{\sqrt{-(\beta^2-\alpha^2)}}{\alpha} \cot\left(\frac{\sqrt{-(\beta^2-\alpha^2)}}{2} \xi \right) \right]. \tag{2.15} $$

If $(\beta^2 - \alpha^2 > 0 \,\&\, \sigma = \alpha)$.

$$ {}_{a}f(\xi) = \left[\frac{-\beta}{\sigma} + \frac{\sqrt{(\beta^2-\alpha^2)}}{\alpha} \tanh\left(\frac{\sqrt{(\beta^2-\alpha^2)}}{2} \xi \right) \right], \tag{2.16} $$

or

$$ {}_{a}f(\xi) = \left[\frac{-\beta}{\sigma} + \frac{\sqrt{(\beta^2-\alpha^2)}}{\alpha} \coth\left(\frac{\sqrt{(\beta^2-\alpha^2)}}{2} \xi \right) \right]. \tag{2.17} $$

If $(\alpha\sigma < 0 \,\&\, \sigma \neq 0 \,\&\, \beta = 0)$.

$$ {}_{a}f(\xi) = \left[\sqrt{\frac{-\alpha}{\sigma}} \tanh\left(\frac{\sqrt{-\alpha\sigma}}{2} \xi \right) \right], \tag{2.18} $$

or

$$ {}_{a}f(\xi) = \left[\sqrt{\frac{-\alpha}{\sigma}} \coth\left(\frac{\sqrt{-\alpha\sigma}}{2} \xi \right) \right]. \tag{2.19} $$

If $(\beta = 0 \,\&\, \alpha = -\sigma)$.

$$ {}_{a}f(\xi) = \left[\frac{-(1+e^{2\alpha\xi}) \pm \sqrt{2(e^{4\alpha\xi}+1)}}{e^{2\alpha\xi}-1} \right], \tag{2.20} $$

or

$$a^{f(\xi)} = \left[\frac{-(1+e^{2\alpha\xi}) \pm \sqrt{e^{4\alpha\xi} + 6e^{2\alpha\xi} + 1}}{2e^{2\alpha\xi}} \right]. \tag{2.21}$$

If $(\alpha = \sigma = 0)$.

$$a^{f(\xi)} = \left[\frac{-(1+e^{2\beta\xi}) \pm \sqrt{2(e^{4\beta\xi} + 1)}}{e^{2\beta\xi} - 1} \right], \tag{2.22}$$

or

$$a^{f(\xi)} = \left[\frac{-(1+e^{2\beta\xi}) \pm \sqrt{e^{4\beta\xi} + 6e^{2\beta\xi} + 1}}{2e^{2\beta\xi}} \right]. \tag{2.23}$$

If $(\beta^2 = \alpha\sigma)$.

$$a^{f(\xi)} = \left[\frac{-\alpha(\beta\xi + 2)}{\beta^2 \xi} \right]. \tag{2.24}$$

If $(\beta = k, \alpha = 2k, \sigma = 0)$.

$$a^{f(\xi)} = \left[e^{k\xi} - 1 \right]. \tag{2.25}$$

If $(\beta = k, \sigma = 2k, \alpha = 0)$.

$$a^{f(\xi)} = \left[\frac{e^{k\xi}}{1 - e^{k\xi}} \right]. \tag{2.26}$$

If $(2\beta = \alpha + \sigma)$.

$$a^{f(\xi)} = \left[\frac{1 - \alpha e^{\frac{1}{2}(\alpha-\sigma)\xi}}{1 - \sigma e^{\frac{1}{2}(\alpha-\sigma)\xi}} \right], \tag{2.27}$$

or

$$a^{f(\xi)} = \left[\frac{\alpha e^{\frac{1}{2}(\alpha-\sigma)\xi} + 1}{-\sigma e^{\frac{1}{2}(\alpha-\sigma)\xi} - 1} \right]. \tag{2.28}$$

If $(-2\beta = \alpha + \sigma)$.

$$a^{f(\xi)} = \left[\frac{e^{\frac{1}{2}(\alpha-\sigma)\xi} + \alpha}{e^{\frac{1}{2}(\alpha-\sigma)\xi} + \sigma} \right]. \tag{2.29}$$

If $(\alpha = 0)$.

$$a^{f(\xi)} = \left[\frac{\beta e^{\beta\xi}}{1 + \frac{\sigma}{2} e^{\beta\xi}} \right]. \tag{2.30}$$

If $(\beta = \alpha = \sigma \neq 0)$.

$$a^{f(\xi)} = \left[\frac{-(\alpha\xi + 2)}{\alpha\xi} \right]. \tag{2.31}$$

If $(\beta = \sigma = 0)$.

$$a^{f(\xi)} = \left[\frac{\alpha}{2} \xi \right] \tag{2.32}$$

If $(\beta = \alpha = 0)$.

$$a^{f(\xi)} = \left[\frac{-2}{\sigma\xi} \right] \tag{2.33}$$

If $(\beta = 0, \alpha = \sigma)$.

$$a^{f(\xi)} = \left[\tan\left(\frac{\alpha\xi + C}{2} \right) \right] \tag{2.34}$$

If $(\sigma = 0)$.

$$a^{f(\xi)} = \left[e^{\beta\xi} - \frac{\alpha}{2\beta} \right] \tag{2.35}$$

Where C is an arbitrary constant.

Step 6: Substituting the above values and the solutions of Eq.(2.5) into Eq. (2.4) weattain the exact solutions of Eq. (2.1).

3. KHATER METHOD IN mKdV EQUATION

The mKdV equation arises in nonlinear optics and in the propagation of long internal waves in a fluid when the coefficient of the usual nonlinear term in the KdV equation, uu_x is zero and the higher order nonlinear term $u^2 u_x$ dominates over higher order dispersive terms. We consider the mKdV equation in the form:

$$u_t + \zeta u^2 u_x + \rho u_{xxx} = 0, \tag{3.1}$$

Where $u = u(x,t)$ and ζ , ρ are non zero constants..

Now, using the wave variable $u(x,t) = u(\xi)$, $\xi = x - vt$, in (3.1) and integrating the resulting equation and neglecting the constant of integration we find

$$-vu + \frac{1}{3}\zeta u^3 + \rho u'' = 0, \tag{3.2}$$

By balancing u^3 and u'', we get $3N = N + 2$, hence $N = 1$. We then suppose the Eq. (3.2) has the following formal solution

$$u(\xi) = a_0 + a_1 a^{f(\xi)}, \tag{3.3}$$

Substituting Eq. (3.3) into Eq. (3.2), we use (2.5). The left-hand side of (3.2) becomes a polynomial in $a^{f(\xi)}$. Setting the each coefficient of different power of $a^{if(\xi)}$, $(i = 0,1,2,3.....)$ to zeroyields a system of algebraic equations.

$$\zeta a_0{}^3 - 3va_0 + 3\alpha\beta\rho a_1 = 0$$

$$\zeta a_0{}^3 + 6\rho\sigma^2 a_1 = 0$$

$$\zeta a_0{}^2 a_1 - va_1 + 2\alpha\rho\sigma a_1 = 0$$

$$\zeta a_0 a_1{}^2 + 3\beta\rho\sigma a_1 = 0$$

Solving the algebraice quations by Maple we get

$$a_0 = \frac{1}{2}\frac{\sqrt{-6\zeta\rho}\beta}{\zeta},\ a_1 = \frac{\sqrt{-6\zeta\rho}\sigma}{\zeta},\ v = \frac{1}{2}\rho(4\alpha\sigma - \beta^2) \tag{3.4}$$

So that, the exact traveling wave solution of Eq.(3.2) is of the form:

$$u(\xi) = \frac{1}{2}\frac{\sqrt{-6\zeta\rho}\beta}{\zeta} + \frac{\sqrt{-6\zeta\rho}\sigma}{\zeta} a^{f(\xi)}, \tag{3.5}$$

which leads to the following travelling wave solutions subjected to the corresponding cases

If $(\beta^2 - \alpha\sigma < 0\ \&\ \sigma \neq 0)$

$$u(x,t) = -\frac{1}{2}\frac{\sqrt{-6\zeta\rho}\beta}{\zeta} + \frac{\sqrt{-6\zeta\rho}}{\zeta}\sqrt{\alpha\sigma - \beta^2}\tan\left(\frac{\sqrt{\alpha\sigma - \beta^2}}{2}(x - vt)\right) \tag{3.6}$$

or

$$u(x,t) = -\frac{1}{2}\frac{\sqrt{-6\zeta\rho}\beta}{\zeta} + \frac{\sqrt{-6\zeta\rho}}{\zeta}\sqrt{\alpha\sigma-\beta^2}\cot\left(\frac{\sqrt{\alpha\sigma-\beta^2}}{2}(x-\nu t)\right) \quad (3.7)$$

If $(\beta^2 - \alpha\sigma > 0 \,\&\, \sigma \neq 0)$.

$$u(x,t) = -\frac{1}{2}\frac{\sqrt{-6\zeta\rho}\beta}{\zeta} - \frac{\sqrt{-6\zeta\rho}}{\zeta}\sqrt{\beta^2-\alpha\sigma}\tanh\left(\frac{\sqrt{\beta^2-\alpha\sigma}}{2}(x-\nu t)\right) \quad (3.8)$$

or

$$u(x,t) = -\frac{1}{2}\frac{\sqrt{-6\zeta\rho}\beta}{\zeta} - \frac{\sqrt{-6\zeta\rho}}{\zeta}\sqrt{\beta^2-\alpha\sigma}\coth\left(\frac{\sqrt{\beta^2-\alpha\sigma}}{2}(x-\nu t)\right) \quad (3.9)$$

If $(\beta^2 + \alpha^2 > 0 \,\&\, \sigma \neq 0 \,\&\, \sigma = -\alpha)$.

$$u(x,t) = \frac{1}{2}\frac{\sqrt{-6\zeta\rho}\beta}{\zeta} + \frac{\sqrt{-6\zeta\rho}\sigma}{\zeta}\left[\frac{\beta}{\alpha} + \frac{\sqrt{\beta^2+\alpha^2}}{\alpha}\tanh\left(\frac{\sqrt{\beta^2+\alpha^2}}{2}(x-\nu t)\right)\right] \quad (3.10)$$

or

$$u(x,t) = \frac{1}{2}\frac{\sqrt{-6\zeta\rho}\beta}{\zeta} + \frac{\sqrt{-6\zeta\rho}\sigma}{\zeta}\left[\frac{\beta}{\alpha} + \frac{\sqrt{\beta^2+\alpha^2}}{\alpha}\coth\left(\frac{\sqrt{\beta^2+\alpha^2}}{2}(x-\nu t)\right)\right] \quad (3.11)$$

If $(\beta^2 + \alpha^2 < 0 \,\&\, \sigma \neq 0 \,\&\, \sigma = -\alpha)$.

$$u(x,t) = \frac{1}{2}\frac{\sqrt{-6\zeta\rho}\beta}{\zeta} + \frac{\sqrt{-6\zeta\rho}\sigma}{\zeta}\left[\frac{\beta}{\alpha} + \frac{\sqrt{-\left(\beta^2+\alpha^2\right)}}{\alpha}\tan\left(\frac{\sqrt{-\left(\beta^2+\alpha^2\right)}}{2}(x-\nu t)\right)\right] \quad (3.12)$$

or

$$u(x,t) = \frac{1}{2}\frac{\sqrt{-6\zeta\rho}\beta}{\zeta} + \frac{\sqrt{-6\zeta\rho}\sigma}{\zeta}\left[\frac{\beta}{\alpha} + \frac{\sqrt{-\left(\beta^2+\alpha^2\right)}}{\alpha}\cot\left(\frac{\sqrt{-\left(\beta^2+\alpha^2\right)}}{2}(x-\nu t)\right)\right] \quad (3.13)$$

If $(\beta^2 - \alpha^2 < 0 \,\&\, \sigma = \alpha)$.

$$u(x,t) = \frac{1}{2}\frac{\sqrt{-6\zeta\rho}\beta}{\zeta} + \frac{\sqrt{-6\zeta\rho}\sigma}{\zeta}\left[-\frac{\beta}{\alpha} + \frac{\sqrt{-\left(\beta^2-\alpha^2\right)}}{\alpha}\tan\left(\frac{\sqrt{-\left(\beta^2-\alpha^2\right)}}{2}(x-\nu t)\right)\right] \quad (3.14)$$

or

$$u(x,t)=\frac{1}{2}\frac{\sqrt{-6\zeta\rho}\beta}{\zeta}+\frac{\sqrt{-6\zeta\rho}\sigma}{\zeta}\left[-\frac{\beta}{\alpha}+\frac{\sqrt{-\left(\beta^2-\alpha^2\right)}}{\alpha}\cot\left(\frac{\sqrt{-\left(\beta^2-\alpha^2\right)}}{2}(x-vt)\right)\right] \tag{3.15}$$

If $(\beta^2-\alpha^2>0\ \&\ \sigma=\alpha)$.

$$u(x,t)=\frac{1}{2}\frac{\sqrt{-6\zeta\rho}\beta}{\zeta}+\frac{\sqrt{-6\zeta\rho}\sigma}{\zeta}\left[-\frac{\beta}{\alpha}+\frac{\sqrt{\left(\beta^2-\alpha^2\right)}}{\alpha}\tanh\left(\frac{\sqrt{\left(\beta^2-\alpha^2\right)}}{2}(x-vt)\right)\right] \tag{3.16}$$

or

$$u(x,t)=\frac{1}{2}\frac{\sqrt{-6\zeta\rho}\beta}{\zeta}+\frac{\sqrt{-6\zeta\rho}\sigma}{\zeta}\left[-\frac{\beta}{\alpha}+\frac{\sqrt{\left(\beta^2-\alpha^2\right)}}{\alpha}\coth\left(\frac{\sqrt{\left(\beta^2-\alpha^2\right)}}{2}(x-vt)\right)\right] \tag{3.17}$$

If $(\alpha\sigma<0\ \&\ \sigma\neq 0\ \&\ \beta=0)$.

$$u(x,t)=\frac{\sqrt{-6\zeta\rho}\sigma}{\zeta}\left[\sqrt{\frac{-\alpha}{\sigma}}\tanh\left(\frac{\sqrt{-\alpha\sigma}}{2}(x-vt)\right)\right] \tag{3.18}$$

or

$$u(x,t)=\frac{\sqrt{-6\zeta\rho}\sigma}{\zeta}\left[\sqrt{\frac{-\alpha}{\sigma}}\coth\left(\frac{\sqrt{-\alpha\sigma}}{2}(x-vt)\right)\right] \tag{3.19}$$

If $(\beta=0\ \&\ \alpha=-\sigma)$.

$$u(x,t)=\frac{\sqrt{-6\zeta\rho}\sigma}{\zeta}\left[\frac{-(1+e^{2\alpha(x-vt)})\pm\sqrt{2(e^{4\alpha(x-vt)}+1)}}{e^{2\alpha(x-vt)}-1}\right] \tag{3.20}$$

or

$$u(x,t)=\frac{\sqrt{-6\zeta\rho}\sigma}{\zeta}\left[\frac{-(1+e^{2\alpha(x-vt)})\pm\sqrt{e^{4\alpha(x-vt)}+6e^{2\alpha(x-vt)}+1)}}{2e^{2\alpha(x-vt)}}\right] \tag{3.21}$$

If $(\beta^2=\alpha\sigma)$.

$$u(x,t)=\frac{1}{2}\frac{\sqrt{-6\zeta\rho}\beta}{\zeta}+\frac{\sqrt{-6\zeta\rho}\sigma}{\zeta}\left[\frac{-\alpha(\beta(x-vt)+2)}{\beta^2(x-vt)}\right] \tag{3.22}$$

If $(\beta = k, \sigma = 2k, \alpha = 0)$.

$$u(x,t) = \frac{1}{2}\frac{\sqrt{-6\zeta\rho}\beta}{\zeta} + \frac{\sqrt{-6\zeta\rho}\sigma}{\zeta}\left[\frac{e^{k(x-vt)}}{1-e^{k(x-vt)}}\right] \tag{3.23}$$

If $(2\beta = \alpha + \sigma)$.

$$u(x,t) = \frac{1}{2}\frac{\sqrt{-6\zeta\rho}\beta}{\zeta} + \frac{\sqrt{-6\zeta\rho}\sigma}{\zeta}\left[\frac{1-\alpha e^{\frac{1}{2}(\alpha-\sigma)(x-vt)}}{1-\sigma e^{\frac{1}{2}(\alpha-\sigma)(x-vt)}}\right] \tag{3.24}$$

or

$$u(x,t) = \frac{1}{2}\frac{\sqrt{-6\zeta\rho}\beta}{\zeta} + \frac{\sqrt{-6\zeta\rho}\sigma}{\zeta}\left[\frac{\alpha e^{\frac{1}{2}(\alpha-\sigma)(x-vt)}+1}{-\sigma e^{\frac{1}{2}(\alpha-\sigma)(x-vt)}-1}\right] \tag{3.25}$$

If $(-2\beta = \alpha + \sigma)$.

$$u(x,t) = \frac{1}{2}\frac{\sqrt{-6\zeta\rho}\beta}{\zeta} + \frac{\sqrt{-6\zeta\rho}\sigma}{\zeta}\left[\frac{e^{\frac{1}{2}(\alpha-\sigma)(x-vt)}+\alpha}{e^{\frac{1}{2}(\alpha-\sigma)(x-vt)}+\sigma}\right] \tag{3.26}$$

If $(\alpha = 0)$.

$$u(x,t) = \frac{1}{2}\frac{\sqrt{-6\zeta\rho}\beta}{\zeta} + \frac{\sqrt{-6\zeta\rho}\sigma}{\zeta}\left[\frac{\beta e^{\beta(x-vt)}}{1+\frac{\sigma}{2}e^{\beta(x-vt)}}\right] \tag{3.27}$$

If $(\beta = \alpha = \sigma \neq 0)$.

$$u(x,t) = \frac{1}{2}\frac{\sqrt{-6\zeta\rho}\beta}{\zeta} + \frac{\sqrt{-6\zeta\rho}\sigma}{\zeta}\left[\frac{-(\alpha(x-vt)+2)}{\alpha(x-vt)}\right] \tag{3.28}$$

If $(\beta = \alpha = 0)$.

$$u(x,t) = \frac{\sqrt{-6\zeta\rho}\sigma}{\zeta}\left[\frac{-2}{\sigma(x-vt)}\right] \tag{3.29}$$

If $(\beta = 0, \alpha = \sigma)$.

$$u(x,t) = \frac{\sqrt{-6\zeta\rho}\sigma}{\zeta}\left[\tan\left(\frac{\alpha(x-vt)+C}{2}\right)\right] \tag{3.30}$$

4. DISCUSSIONS ON RESULT

Different types of solutions of the mKdV equation obtained by Chen et al [28] using double sub-equation method, Triki and Wazwaz [29] applying sub-ode method, Wanget al [30] implementing the (G'/G)-expansion method, Yan [31] utilizing the extended-tanh function method, Smyth and Worthy [32]employing the approximate method, Biswas et al [33]using Lie symmetry method. But implementing the suggested Khater methoda huge amount of travelling wave solutions of mKdV equation have been generated, which consist of trigonometric function, hyperbolic function, exponential function and rational forms. We realize that our established solutions of mKdV equation contains the solutions found by the aforesaid methods as well as different new solutions, which obvious that Khater method is more general one.

5. CONCLUSIONS

We have studied the mKdV equation using Khater method. And established a rich class of general new exact travelling solutions, comparing the existing solutions. The new found solution might be significant to analyze the complicated phenomena arising in the areas of physics, engineering and other physical sciences and able to expand the influence on future research work. Moreover, our study shows that Khater method is effective, consistent, straightforward and practically well suited in finding exact traveling solutions as well as different types of partial differential equations.

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